# STRUCTURE OF A SHOCK-WAVE FRONT IN QUARTZ IN THE REGION OF THE PHASE TRANSITION OF QUARTZ INTO STISHOVITE

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Shock waves provide one of the most powerful means for studying the properties of materials at high pressures. Conspicuous among the materials investigated are those which undergo polymorphic phase transformations when acted upon by shock waves. The study of the kinetics of such transformations is both of interest in itself and useful in applications to the problem of determining the properties of high-pressure phases, since the phase composition of the final states behind a shock wave is often a nonequilibrium state owing to the special features of the kinetics of the transformation. Up to now, researchers have accumulated a fairly large amount of experimental data on phase transformations in shock waves, and a survey and analysis of these is given in [1]. Despite these advances, it must be recognized that the physical picture, even in its main outlines, is still not sufficiently clear, while the theory of the phenomenon is in the very earliest stages of its development. The reason for this lies in the extraordinary methodological difficulties of the experiment with which a researcher is confronted when he tries to penetrate into the details of the structure of a shock-wave front. The traditional shock-wave method is based on a study of the final result of the shock loading and yields very little information in the details of the transition process. Under these circumstances, we consider it helpful to use the opposite approach as well – to construct physically realistic models and investigate them numerically in order to clarify the characteristic features of the phenomenon that can be experimentally verified. Of course, today it is impossible to construct a complete microscopically adequate theory; it must necessarily be semiempirical, with numerical parameters which can be determined only roughly. Subsequent comparison with experimental results can help in correcting both the model and the values of the numerical parameters. As the object of our investigation, we selected quartz in the region of its transition into stishovite. The choice of quartz was not an accident. The reason for it is that experimental material already available is capable of casting some light on the nature of the transition kinetics and severely restricting the range of choice in the construction of the model for the phenomenon. One such fundamental fact is the constant velocity of the shock wave without nonstationary doubling of the front in the region of the phase transition. This property makes the transition of quartz into stishovite (together with the transition of graphite into diamond) substantially different from the phase transformations of other materials in shock waves.

1. **The Physical Model. The Determining System of Equations.** In [2] we advanced the hypothesis that the phase transition of quartz into stishovite in a shock wave takes place through a martensitic mechanism. Today it is recognized that this mechanism is fundamental in shock-wave polymorphic transitions at relatively low pressures [1]. However, in the case of quartz the martensitic mechanism does not appear in its pure form. The essence of this action is that a reaction is produced and maintained by a fairly intensive field of shearing stresses which exists in the wave front. In support of this hypothesis, we can advance two weighty arguments: in the first place, in static experiments on the phase transformation of quartz, where there are no shearing stresses, we do not find a martensitic type of reaction; in the second place, the shearing stress is the only parameter which decreases discontinuously when single compressibility is replaced by double compressibility, with the formation of a nonstationary two-wave configuration. In the first wave, behind its front, the shearing stresses relax and the second wave is propagated in the medium with isotropic pressure. It should be borne in mind that the reason the shock-wave velocity remains constant without doubling is precisely this sharp variation of the stishovite concentration when single compression is replaced by double [3]. It should be noted that the role of the shearing stress is twofold: On the one hand, its action gives rise to a multiplication of lattice defects which serve as possible nucleation centers, and on the other hand, the growth of the nucleus is itself undoubtedly caused by a shearing-stress field which is external to it. If the latter were not true, then under statistical conditions the martensitic mechanism would be operative, since in any real specimen there are always enough defects. Furthermore, in a doubled shock wave there would be no slowing down of the reaction, since unlike the shearing stress, the number of defects depends only slightly on whether we have one or two waves passing through the material.

As was noted in [2], the necessity of the existence of a shearing stress for the quartz–stishovite transition may be due to the fact that the characteristic deformation of the transition has no invariant plane, and the resulting disorientation may be eliminated either by plastic shear or by rotation of the block as a whole. This is what distinguishes in principle the quartz–stishovite system and similar systems from others in which the lattices of the phases are well adjusted and a martensitic reaction does not require any external shearing stress. The martensitic reaction in systems of the quartz-stishovite type

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may be distinguished from the general mass of similar reactions, and by reason of its characteristic criterion, we may call it tensogenic.

Proceeding from the foregoing, we can formulate the first fundamental requirement for the construction of a model of the phenomenon. This requirement is that the medium can be described as elastoplastic (we shall consider it isotropic), with the inclusion of a relaxation of shearing stresses based on the dynamics of dislocations. Thus, the scheme must include equations which describe the motion and multiplication of dislocations. With regard to the last fact, we cannot at this point go into details concerning the operation of the dislocation sources but must use an empirical relation for the dislocation density as a function of the plastic-shear value [4, 5]. The total deformation must be divided into an elastic cubic compression, an elastic shear, and a plastic shear. Another characteristic feature of the model is the use of the theory of finite deformations. The reason for this is that, in the first place, on an elastic front the elastic shear is finite, and as the relaxation of shearing stresses proceeds, the plastic shear becomes finite (of the order of the cubic compression). Finite deformations are introduced by a scheme similar to that of [6, 7]. The model is completed by the kinetic equation for determining the phase concentrations and their equations of state. We shall consider one-dimensional plane flow and choose the coordinate axes in such a way that the x-axis will be directed along the flow, while the other two axes are perpendicular to it. These directions will be the principal axes of the deformation and stress tensors.

The matrices for the total deformation F, the elastic deformation $F^e$, and the plastic deformation $F^p$ have the following form:

$$F=\begin{pmatrix}\lambda_1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix},\quad F^e=\begin{pmatrix}\lambda_1^e & 0 & 0\\ 0 & \lambda_2^e & 0\\ 0 & 0 & \lambda_2^e\end{pmatrix},\quad F^p=\begin{pmatrix}\lambda_1^p & 0 & 0\\ 0 & \lambda_2^p & 0\\ 0 & 0 & \lambda_2^p\end{pmatrix}.$$

From the condition $F = F^e F^p$ we obtain the relations

$$\lambda_1=\lambda_1^p\lambda_1^e,\quad \lambda_2^p\lambda_2^e=1.$$

The condition that the plastic deformation does not change the volume can be reduced to

$$\lambda_1^p\left(\lambda_2^p\right)^2=1.$$

It is convenient to introduce new independent variables instead of these: the cubic compression

$$\delta=(\lambda_1)^{-1}$$

and the elastic pure shear

$$\xi^e=\lambda_1^e\left(\lambda_2^e\right)^{-1}.$$

The elastic pure shear can be expressed in the form

$$\xi^p=\lambda_1^p\left(\lambda_2^p\right)^{-1}=\left(\delta\xi^e\right)^{-1}.$$

We introduce the principal values of the tensor for the stresses $\sigma_1$ and $\sigma_2 = \sigma_3$. We shall assume that the spherical part of $\sigma_{ik}$ – the cubic pressure

$$p=(1/3)(\sigma_1+2\sigma_2)$$

at temperature T = 0 (cold pressure $p_{cold}$) depends only on the compression $\delta$: $p_{cold} = p_{cold}(\delta)$. The form of this function in the nonlinear case, generally speaking, is arbitrary and can be determined separately. We shall consider the deviator part of $\sigma_{ik}$ (the maximum shearing stress) and denote it by $\tau$:

$$\tau=(1/2)(\sigma_1-\sigma_2).$$

The deviators of the stress and deformation tensors are interrelated. For infinitesimal transformations this relation is given by a linear Hooke's law. We introduce the matrix of an infinitesimal elastic deformation from the state whose deformation is given by the matrix $\{\lambda^e\}$ to the state $\{\lambda^e + d\lambda^e\}$. The desired matrix will be $\{1 + d\lambda^e/\lambda^e\}$. The deformation tensor will be given by the matrix $\{d\lambda^e/\lambda^e\}$, and Hooke's law can be written in the form

$$d\tau=-\mu\left(\frac{d\lambda_1^e}{\lambda_1^e}-\frac{d\lambda_2^e}{\lambda_2^e}\right)=-\mu\frac{d\xi^e}{\xi^e}$$

for $\delta$ = const, i.e.,

$$\partial\tau/\partial\xi^e=-\mu/\xi^e. \tag{1.1}$$

Here the shear modulus $\mu$ is, in general, a function of the compression and the elastic shear. Essentially, (1.1) is the definition of the shear modulus, and its physical meaning becomes clear when we make an additional assumption concerning the function $\mu(\delta, \xi^e)$. An essential restriction on the form of this function can be obtained as follows. We calculate the work

of the elastic forces of deformation of the body. The work done by the body when we change $\lambda_i$ by $d\lambda_i$, referred to a unit mass of the body, will be

$$dA = (1/\rho_0)(\sigma_1\lambda_2\lambda_3 d\lambda_1 + \sigma_2\lambda_1\lambda_3 d\lambda_2 + \sigma_3\lambda_1\lambda_2 d\lambda_3).$$

Using the properties of the deformation being considered, we finally obtain

$$dA = -\frac{1}{\rho^2} p_x(\rho)\, d\rho + \frac{4}{3}\frac{1}{\rho}\tau\left(\frac{d\xi^e}{\xi^e} + \frac{d\xi^p}{\xi^p}\right).$$

Thus, the total work is equal to the sum of the elastic work of the cubic compression, the work of the elastic shear, and the work of the plastic shear. Now we determine the internal energy as a function of the deformations from the first law of thermodynamics, omitting the plastic deformation, since it is irreversible. It was found experimentally that not all of the energy of the plastic deformation goes into heat; about 10% of it goes into the latent energy of the elastic stresses of the newly generated dislocations, but we shall disregard this part. Then, for the change in the internal elastic energy we obtain

$$dE^e = \frac{1}{\rho^2} p_x(\rho)\, d\rho - \frac{4}{3}\frac{\tau}{\rho}\frac{d\xi^e}{\xi^e}.$$

From the fact that $dE^e$ is a total differential, it follows that

$$\frac{\partial}{\partial\rho}\left(\frac{\tau}{\rho}\right) = 0.$$

Together with (1.1), this yields

$$\mu = \delta\varphi(\xi^e),$$

where $\varphi(\xi^e)$ is an arbitrary function of the elastic shear. Since in our case the elastic shear is almost always small, we shall assume that $\varphi(\xi^e)$ is constant, after which we have $\mu = \mu_0\delta$, $\tau = -\mu_0\delta \ln \xi^e$, if we take $\tau = 0$ when $\xi^e = 1$.

In order to simplify the model, we shall describe our two-component medium by a single shear modulus. In a more exact theory, we should average the moduli of the components, both the shear moduli and the cubic-compression moduli affecting each other, since the cubic compression of a specimen is accompanied by shears of the components in order to maintain the continuity, and vice versa. To the elastic pressure and the energy, we add the thermal terms, and we assume that we have additivity of volume, internal energy, and entropy of the components for equal temperatures and pressures. The kinetics of the plastic deformation will be described by Orovan's equation

$$\frac{1}{\xi^p}\frac{d\xi^p}{dt} = -bNv, \tag{1.2}$$

where b is the Burgers vector, N is the density of dislocations, and v is their velocity.

In the case of small deformations, we usually consider b and N independent of the compression; in our case this functional relation must be taken into account. We introduce the quantities $b_0$ and $N_0$, referred to an uncompressed unit volume; then $b = b_0\delta^{-1/3}$, $N = N_0\delta^{2/3}$ (the dislocations are frozen into the lattice). Now we rewrite (1.2) in the form

$$\frac{1}{\xi^p}\frac{d\xi^p}{dt} = -b_0 N_0 \delta^{1/3} v.$$

The derivative with respect to time should be taken in Lagrange's sense. The variation of the dislocation velocity as a function of the shearing stress will be described by Gilman's formula

$$v = c_t \exp(-\tau_0/\tau), \tag{1.3}$$

where $c_t$ is the transverse velocity of sound and $\tau_0$ is a constant parameter.

For the dislocation density we take a linear variation as a function of the plastic deformation

$$N_0 = N_{00} + k \ln \xi^p. \tag{1.4}$$

In our model we assume that the relaxation of the shearing stresses takes place only as a result of the conservative motion of slip dislocations. In the exact theory we should also take account of the relaxation resulting from the phase transition. If we assume that the transition is stimulated by the shearing stress, then, according to Le Chatelier's principle, it should lead to a decrease in $\tau$.

The last point to be considered in the model is the kinetic equation of the phase transformation. We consider a Lagrangian unit element of volume. Then there will be $dN_0/dt$ dislocations generated in it per unit of time. However, a dislocation, being a linear object, cannot serve as a center of nucleation. Such a center may be, for example, the intersection of dislocations; therefore, we introduce the parameter $l_0$, the average distance between active nodes of the

dislocation network. Then $(1/\rho_0 l_0)dN_0/dt$ centers will be generated per unit of time in a gram of material. From the total number we must subtract those which belong to the unstable phase. Assuming that the defects are uniformly distributed, we can find this fraction. To do this, we use the relations

$$V_1 + V_2 = V, \quad V_1\rho_1 + V_2\rho_2 = 1, \quad \beta\rho_1 + (1-\beta)\rho_2 = 1/\rho,$$

where $\beta$ is the weight concentration of the first (light, unstable) phase. We find that the fraction of the volume $V_1/V = \beta\rho/\rho_1$ and the rate of generation of centers in the light phase is equal to $(\beta\delta/l_0\rho_1)dN_0/dt$.

Crystals of the new phase begin to grow on the centers thus formed. We denote the time of generation by t′ and denote the mass of the crystal at time t by m(t′, t). Now in order to find the concentration of the heavy phase, $1-\beta$, we must integrate with respect to the generation time t′ the entire mass of the new phase in a gram of material

$$1-\beta(t) = \frac{1}{l_0}\int_{t_0}^{t} \frac{\beta(t')}{\rho_1(t')}\frac{dN_0}{dt'} m(t', t)\, dt' \tag{1.5}$$

($t_0$ is the time at which the critical conditions for the start of the transition are reached). The integration is carried out along a streamline. It remains now to determine the form of the function m(t′, t). We assume that the shape of the growing crystal is the same as in an equilibrium martensitic crystal, namely, a double-convex lens [8]; then its volume is proportional to $R^{5/2}$, where R is the radius of the rim of the lens. We introduce the rate of growth $v_1$ of the lens radius; then

$$R(t', t) = \int_{t'}^{t} v_1(t'')\, dt''.$$

Here we must make an assumption concerning the rate of growth $v_1$ of the crystal, since we do not have sufficient experimental data. All we know is that the rate of growth of a martensitic crystal, like the velocity of the dislocations, may take on a value equal to the transverse velocity of sound. This is natural, since the restructuring of the lattice at the boundary takes place through motion along the transition dislocation boundary. Starting from these rough ideas, we assume that the velocity $v_1$ is qualitatively the same kind of function of the shearing stress (necessary in the case of a tensogenic transition) as in the case of slip dislocations (1.3). The only difference is that we introduce into it a threshold with respect to $\tau$:

$$v_1 = c_t \exp\left(-\frac{\tau_1}{\tau - \tau_2}\right).$$

For $\tau \leqslant \tau_2$ we set $v_1 = 0$. We must also set $v_1 = 0$ in the region of thermodynamic stability of the light phase. If the shock wave has sufficient intensity, the reaction begins at once on the front, with generation on the defects in the motionless material (the term $N_{00}$ in (1.4)). In the kinetic equation (1.5) there must be corresponding to this a term on the right side equivalent to a $\delta$-type source with a generation time equal to the time of arrival of the shock wave, i.e., $dN_0/dt'$ includes a term $2N_{00}\delta(t' - t_0)$ . Now we write the complete system of equations in Lagrangian variables:

the equation of continuity and determination of velocity

$$\partial x/\partial a = 1/\rho, \ \partial x/\partial t = u,$$

the Euler equation

$$\partial u/\partial t + \partial\sigma_1/\partial a = 0,$$

the entropy equation

$$\frac{\partial S}{\partial t} = \frac{1}{T}(\Phi_2 - \Phi_1)\frac{\partial\beta}{\partial t} + \frac{4}{3}\frac{1}{T}\frac{1}{\rho}\frac{\tau}{\xi^p}\frac{\partial\xi^p}{\partial t}$$

(this first term gives us the growth of the entropy as a result of the nonequilibrium phase transition [2], the second that caused by the work of plastic deformation, where the $\Phi_k$ are the Gibbs thermodynamic potentials),

the equation of state

$$V = \beta V_1(p, T) + (1-\beta)V_2(p, T),$$
$$E = \beta E_1(p, T) + (1-\beta)E_2(p, T), \quad S = \beta S_1(p, T) + (1-\beta)S_2(p, T)$$

(the equations of state of individual phases will be given below),

the definition of pressure

$$p = (1/3)(\sigma_1 + 2\sigma_2),$$

Hooke's law for shear

$$\sigma_1 - \sigma_2 = -2\mu_0\delta \ln \xi^e,$$

the connection between shear and cubic compression

$$\xi^p\xi^e\delta = 1,$$

the kinetics of plastic flow

$$(1/\xi^p)\partial\xi^p/\partial t = -b_0 N_0 \delta^{1/3} v,$$
$$N_0 = N_{00} + k \ln \xi^p, \quad v = c_t \exp(-\tau_0/\tau),$$

the kinetics of phase transformation

$$1 - \beta = B\frac{\rho_{20}\beta_F}{\rho_0} N_{00} R^{5/2}(t_0, t) + Bb_0\rho_{20}k\int_{t_0}^{t} \beta(t')\delta^{4/3}(t')\rho_1^{-1}(t')\left(N_{00} + k\ln\xi^p\right) vR^{5/2}(t', t)\, dt',$$

$$R(t', t) = \int_{t'}^{t} v_1(t'')\, dt'', \quad v_1 = c_t \exp\left(-\frac{\tau_1}{\tau - \tau_2}\right).$$

Here $\beta_F$ is the concentration of the light phase on the wave front; B is a proportionality factor, a parameter which can be varied. We write out the boundary conditions. On the piston $u = u_0$. On the elastic front of the shock wave we have an absence of plastic shear $\xi_F^p = 1$, with the presence only of the light phase $\beta_F = 1$ and exact Hugoniot boundary conditions.

**2. Numerical Calculations and Their Results.** For the numerical integration of the system of equations, we set up a special program whose main distinguishing features were the use of the method of characteristics and exact boundary conditions on the elastic front of the shock wave. The equations of state of the phases were taken in the form of sums of cold and thermal terms. The cold pressure was taken in the form

$$p_X(\rho) = \frac{\rho_0 c_0^2}{n}\left[\left(\frac{\rho}{\rho_0}\right)^n - 1\right].$$

where $\rho_0$ is the initial density; $c_0$ is the initial volumetric velocity of sound. The ratio of the thermal pressure $p_T$ to the volumetric thermal energy (the Gruneisen coefficient $\Gamma$) is assumed to be constant. The thermal energy (per gram), $E_T$, is proportional to the temperature T, with a constant proportionality factor $c_V$. The total energy which appears in the Hugoniot equation for the shock-wave adiabatic $E = (1/2)\sigma_1(1/\rho_0 - 1/\rho)$, can be written in the form

$$E = E_0 + \int_{\rho_0}^{\rho} p_X(\rho)\frac{d\rho}{\rho^2} + \frac{2}{3}\frac{\mu_0}{\rho_0}(\ln\xi^e)^2 + E_T.$$

The entropy can be found by integrating the equation

$$TdS = dE_T + p_T\, dV.$$

We used the following values of the constant parameters:

quartz

$$\rho_0 = 2.65\ \text{g/cm}^3, c_0 = 3.7\ \text{km/sec}, = 6, \Gamma = 0.653, c_V = 1.25\cdot10^7\ \text{erg/g}\cdot\text{deg},$$
$$E_0 = 0,\ S_0 = 0;$$

stishovite

$$\rho_0 = 4.29\ \text{g/cm}^3, c_0 = 7.6\ \text{km/sec}, = 3, \Gamma = 1.2, c_V = 1.25\cdot10^7\ \text{erg/g}\cdot\text{deg},$$
$$E_0 = 6.6\cdot10^9\ \text{erg/g}, S_0 = -2.6\cdot10^6\ \text{erg/g}\cdot\text{deg},$$

Here $S_0$ is the entropy in the normal state; the entropy and energy of the quartz in the normal state were assumed to be zero. The initial shear modulus is $\mu_0 = 0.2$ Mbar. The modulus of the Burgers vector is $b_0 = 5\cdot10^{-8}$ cm. The parameters in the dislocation density were $N_{00} = 2\cdot10^8\ \text{cm}^{-2}$, $k = 10^{12}\ \text{cm}^{-2}$. The parameters in the dislocation velocities were $\tau_0 = 30$ kbar, $\tau_1 = 30$ kbar, $\tau_2 = 15$ kbar.

The threshold $\tau_2 = 15$ kbar was selected from the condition that at the first break-point of the shock-wave adiabat of quartz (and below it) the reaction should be impossible. In a stationary wave with an amplitude of p = 140 kbar at the plastic tail, the parameters of the equations of state match the known data on the dynamic compressibility of quartz and stishovite, except for one point which we shall discuss in more detail. In our case there is a decrease in the shear modulus in comparison with the experimental value and an increase in the curvature of the cold pressure of the quartz. The reason is the following. In the general case the position of the elastic shock-wave adiabat with respect to the wave ray (in the plane $\sigma_1$, V) going from the point of the initial state to the first breakpoint can be arbitrary, and in particular it can be such that the wave ray will lie entirely below the elastic shock-wave adiabat. For quartz this does in fact happen,

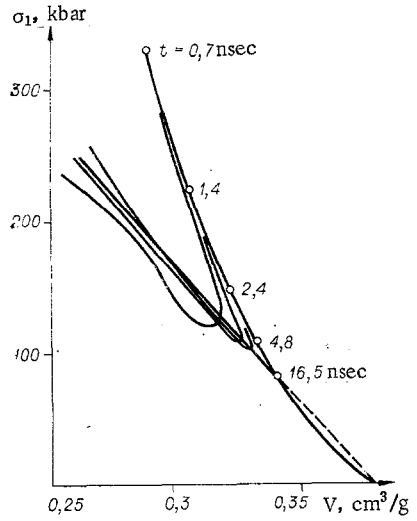


Fig. 1

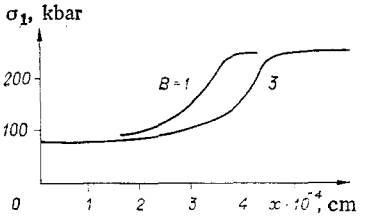


Fig. 2

and thus, the velocity of the shock wave at the break-point is D = 5.7 km/sec, while the longitudinal velocity of sound is $c_l$ = 6 km/sec. A consequence of this will be a constant overtaking of the elastic wave by the plastic wave, i.e., in front of the plastic wave there will be propagated an elastic precursor with constantly decreasing amplitude. The presence of an elastic precursor does not change anything essential in our problem, but it is very inconvenient for numerical calculation, since it includes an ever-larger number of calculation points, so that the entire available computer capacity is used up in calculating this quantity alone. In our case the wave ray intercepts the elastic shock-wave adiabat. In this case the total picture of the flow will be stationary in the limit, which means that we can manage with a constant number of calculation points. In the calculations we used 50 points. The constant factor in the formula for the kinetics of the transition was varied in order to show that for a sufficiently "strong" type of kinetics the mechanism establishing a constant wave velocity will be operative. In order to save machine time, this variation was carried out as follows. First we carried out the calculation with a minimum value of B = 1, and the calculation was continued until a stationary profile was established. Next we substituted larger values. Since the stationary profiles for different values of B were no longer very different from one another, each new constant value was reached rapidly. We carried out similar calculations with varied values of B for two values of the boundary velocity: $u_0$ = 1.371 km/sec and 1.698 km/sec. The final states in both cases lie above the break-point, in the region of phase mixing. The process of establishing a steady state can conveniently be illustrated on the graph of $\sigma_1$ as a function of V. In a stationary profile the points at which the profile is cut should lie on a single straight line, while in a nonstationary profile with a two-wave configuration they should lie on two straight lines. In Fig. 1 we show a family of curves for the case $u_0$ = 1.698 km/sec, B = 1. We can see the establishment of one stationary front. The total time for establishing this is $\sim 1 \cdot 10^{-8}$ sec. It should be noted that the functions we chose for the velocities of the dislocations tend exponentially to zero, and therefore a formally rigorous stationary condition is reached only asymptotically as $t \to \infty$, but even at times of the order of those indicated, the profiles differ only slightly from the limiting cases. Both of the steady-state solutions (for different values of $u_0$), from B = 1 to B = 4, lie practically on the same wave ray. For $u_0$ = 1.371 km/sec, D = 5.63 km/sec, for $u_0$ = 1.698 km/sec, D = 5.66 km/sec, i.e., the values of D agree within the limits of our accuracy.

Figure 2 shows a profile of $\sigma_1(x)$ in a coordinate system in which the front is at rest (for the front x = 0) for the case $u_0$ = 1.698 km/sec. It can be clearly seen that the front consists of a stationary sequence of two fronts: an elastic shock-wave front with a practically constant flow behind it and, at a fixed distance from it, a plastic-wave front on which the phase transition takes place. This distance is determined by the action of the stability mechanism. The second wave finds itself a location such that the integral determining the concentration of stishovite (1.5) (in the stationary profile the integral with respect to t can be replaced with an integral with respect to x) will have a given value such that the final state will lie on the wave ray. If, for example, the distance between the waves is reduced, there will be an increase in the amount of stishovite produced and a decrease in the velocity of the second wave, after which the distance between the two waves will increase, and vice versa. From this point of view it is clear that the more intensive the kinetics, the greater must be the distance between the elastic and plastic waves. This is confirmed by the numerical calculation. In Fig. 2 we show the profiles with B = 1 and B = 3, which clearly illustrate this fact.

Interesting problems arise in connection with the possibilities of experimentally verifying the results of the calculation. It would be desirable to establish the presence of a stationary two-wave structure. Unfortunately, this problem is very difficult if the distances and times correspond to the calculated ones; however, as already noted, the kinetic constants may differ from those adopted, and a study of the front structure may be possible.

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# THE PROCESS OF SPALL FRACTURE


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**UDC 532.593**


Many investigations of spall phenomena after the emergence of the compression pulse to the free surface of a specimen show that the strength value realized in spalling depends on the characteristic time of action of the load. A number of studies [1-4] propose discrete criteria for spall fracture which determine the possibility of failure in terms of the value of the tensile stress and the time during which it acts at a particular cross section of the specimen. However, on the one hand, the load at any cross section may, in general, vary arbitrarily, and on the other hand, the failure process itself leads to a drop in the tensile stress, which makes the actual application of the discrete spall criteria difficult. The authors of [5-7] discuss the possibility of introducing a continuous measure of failure into the spall criterion; such a measure may be the dimensions and number of the cracks in the specimen, the residual strength of a half-ruptured specimen, etc. Experimental information on the failure process can be obtained from a metallographic analysis of preserved specimens [5, 6], or from experiments on the continuous recording of the velocity of the free surface of the specimen when a compression pulse and a "spall" pulse emerge onto it [8-11]. It is impossible at the present time to obtain continuous quantitative information directly from the failure zone.

In the present article we consider the effect of the kinetics of failure on the gas dynamics of a wave process. In the gas-dynamic analysis of a phenomenon, it is most convenient to use the specific volume of a crack, $v_{cr}$, as the measure of the failure. The shear strength of the medium will be disregarded in what follows. The rate of growth of the cracks (or pores), as can be deduced from general considerations [6, 7], is determined by the value of the negative pressure p acting on the material and by the degree of failure achieved, $v_{cr}$:

$$\dot{v}_{cr} = f(p,\ v_{cr}). \tag{1}$$

Barbee et al. [6] have proposed specific expressions for the failure kinetics of (1), which are based on a model of exponential generation and ductile growth of the cracks.

In order to see what kind of information concerning the effect of continuous failure on the gas dynamics of the process can be obtained in general form, we shall follow the change of state of a substance along the characteristics in a linear material, i.e., in a material whose equation of state has the form

$$\frac{\rho^2}{\rho_0^2}\left(\frac{\partial p}{\partial \rho}\right)_{v_{cr}=\text{const}} = a^2 = \text{const}, \tag{2}$$

where $\rho$, $\rho_0$ are the instantaneous and initial values of the density of the substance. The specific volume of the failed medium, v, consists of the volume of the solid material, $v_{sol}$, and the volume of the cracks, $v_{cr}$:

$$v = v_{sol} + v_{cr}. \tag{3}$$

The equations of motion and continuity, taking account of (1)-(3), for a one-dimensional case have in Lagrangian coordinates the form

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